

EFFECTS OF SUBSTRATE ANISOTROPY ON THE DISPERSION
OF TRANSIENT SIGNALS IN MICROSTRIP LINES

G. W. Zheng*, K. S. Chen* and S. T. Peng**

* Zhejiang University, Hangzhou, 310008, P.R. China

** New York Institute of Technology, Old Westbury, New York

ABSTRACT

The analysis of dispersion characteristics of transient signal in microstrip lines with anisotropic substrate is developed here, with particular attention directed toward the effects of arbitrary orientations of the principal optical axis in anisotropic substrates. Numerical simulations are carried out for the propagation of transient signals, square or Gaussian pulses, along microstrips with anisotropic substrates. It is shown that the dispersion characteristics is substantially affected by the change of the orientation angle of the principal optical axis in the substrate.

I. INTRODUCTION

With the development of MIC and MMIC techniques, anisotropic materials such as GaAs and sapphire have been used as substrates in many microwave and millimeter wave components. Effects of substrate anisotropy on the dispersion characteristics of transient signals in transmission lines have attracted considerable attention in the past. However, so far the simulation of the evolution of the transient signal, square or Gaussian pulses, propagating on microstrips has not been completely performed when the principal optical axis of anisotropic substrates is arbitrarily oriented. In this paper, we try to develop a method to treat the problem with microcomputer. To this end,

the numerical simulation is carried out alternatively in both time domain and frequency domain, and a modified 2D FD-TD method is proposed. At first, the transient signal at the initial position $z=0$ is transformed into frequency domain, then the phase shift from initial position to $z=1$ is calculated for all major frequency components, and by taking the inverse transform the time domain representation of the transient signal at $z=1$ is finally obtained. It should be noted that the eigenvalue problem is solved in time domain with the modified 2D FD-TD method where the Maxwell's equation is expressed by a set of difference equations about \vec{D} , \vec{E} and \vec{H} rather than about \vec{E} and \vec{H} , as commonly used. The formulation will be given in section II and the results of numerical simulations in section III. It is shown that the distortion of transient signals propagating along microstrip lines is substantially affected by the change of orientation angle of principal optical axis in the substrate.

II. FORMULATION

For simplicity, the materials, both metal and dielectric, are assumed to be lossless and the metal strips to be zero-thickness in present analysis. The geometry of substrates under consideration is shown in Fig.1. The dielectric substrate is generally expressed by a permittivity tensor

$$\epsilon = \epsilon_0 \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix} \quad (1)$$

where ϵ_0 is the permittivity in free space.

Suppose a transient signal, either voltage or electric field, at the initial position $z=0$ can be represented by

$$v(t, z=0) = \begin{cases} v(t) & -T/2 \leq t \leq T/2 \\ 0 & \text{elsewhere} \end{cases} \quad (2)$$

In the frequency domain, the signal can be written as $V(\omega, z=0)$, where $V(\omega)$ and $v(t)$ constitute a transform pair. At the position of $z=1$ the signal (or pulse) in the frequency domain becomes

$$V(\omega, z=1) = V(\omega, z=0) \exp[-j\beta(\omega)1] \quad (3)$$

where $\beta(\omega)$ is the phase constant and the frequency-dependant attenuation constant $\alpha(\omega)$ is assumed negligible. By taking the inverse transform of (2), the representation of the pulse at the position of $z=1$ in time domain is obtained and can be written as

$$v(t, z=1) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} V(\omega, z=0) \exp[j(\omega t - \beta(\omega)1)] d\omega \quad (4)$$

The phase constant $\beta(\omega)$ can be expressed in terms of effective dielectric constant $\epsilon_{\text{eff}}(\omega)$,

$$\beta(\omega) = \omega/c\sqrt{\epsilon_{\text{eff}}(\omega)} \quad (5)$$

The expression $V(\omega, z=0)$, the transform of $v(t, z=0)$, can be easily obtained for many common waveshapes, such as the square and the Gaussian DC pulse or any RF waves modulated by these two kinds of pulses.

Many methods have been proposed to determine the eigenvalue $\beta(\omega)$ or $\epsilon_{\text{eff}}(\omega)$ for microstrip. However, when the principal axis of anisotropic substrate is arbitrarily oriented, no general treatment is available so far, and in this paper we put forward a modified 2D FD-TD method to approach the problem.

The Maxwell's equation is written as

$$\nabla \times \vec{E} = -(\partial \vec{H} / \partial t) / \mu \quad (6a)$$

$$\vec{\partial D} / \partial t = \nabla \times \vec{H} \quad (6b)$$

where

$$\vec{D} = \vec{\epsilon} \vec{E} \quad (7)$$

Since the structure is uniform in z -direction, the field components can be expressed in the form

$$\begin{aligned} H_x &= h_x(x, y) \exp(-j\beta z) \\ H_y &= h_y(x, y) \exp(-j\beta z) \\ H_z &= j h_z(x, y) \exp(-j\beta z) \\ D_x &= j d_x(x, y) \exp(-j\beta z) \\ D_y &= j d_y(x, y) \exp(-j\beta z) \\ D_z &= d_z(x, y) \exp(-j\beta z) \\ E_x &= j e_x(x, y) \exp(-j\beta z) \\ E_y &= j e_y(x, y) \exp(-j\beta z) \\ E_z &= e_z(x, y) \exp(-j\beta z) \end{aligned} \quad (8)$$

where β is the wavenumber in z -direction. In this way, the three-dimensional problem can be simplified to a two-dimensional one, as will be shown below.

Substituting (8) into (6) we obtain

$$\begin{aligned} \partial h_x / \partial t &= (\beta e_y - \partial e_z / \partial y) / \mu \\ \partial h_y / \partial t &= (-\beta e_x + \partial e_z / \partial z) / \mu \\ \partial h_z / \partial t &= (\partial e_x / \partial y - \partial e_y / \partial x) / \mu \\ \partial d_x / \partial t &= \partial h_z / \partial y + \beta h_y \\ \partial d_y / \partial t &= -\partial h_z / \partial x - \beta h_x \\ \partial d_z / \partial t &= \partial h_y / \partial x - \partial h_x / \partial y \end{aligned} \quad (9)$$

Following Yee's work^[1], known as FD-TD method, each of these scalar equations can be expressed in finite-difference forms. With Yee's nomenclature any function of space (2-dimensional in this paper) and time is discretized

$$F^n(i, j) = F(i \Delta x, j \Delta y, n \Delta t) \quad (10)$$

where $\Delta x = \Delta y = \Delta l$ is the space increment and Δt is the time one. By positioning the components of d , e , and h on the mesh as depicted in Fig. 2 and evaluating d , e , and h at alternate half time steps, we obtain the components of Maxwell's equations

$$\begin{aligned} h_x^{n+1/2}(i, j+1/2) &= h_x^{n-1/2}(i, j+1/2) + \\ \Delta t / \mu [\beta e_y^n(i, j+1/2) - (e_z^n(i, j+1) - e_z^n(i, j)) / \Delta y] \end{aligned}$$

$$\begin{aligned}
h_y^{n+1/2}(i+1/2, j) &= h_y^{n-1/2}(i+1/2, j) + \\
\Delta t / \mu [(e_z^n(i+1, j) - e_z^n(i, j)) / \Delta x - \beta e_x^n(i+1/2, j)] \\
h_z^{n+1/2}(i+1/2, j+1/2) &= h_z^{n-1/2}(i+1/2, j+1/2) \\
&+ \Delta t / \mu [(e_x^n(i+1/2, j+1) - e_x^n(i+1/2, j)) / \Delta y \\
&- (e_y^n(i+1, j+1/2) - e_y^n(i, j+1/2)) / \Delta x] \\
d_x^n(i+1/2, j) &= d_x^{n-1}(i+1/2, j) \\
&+ \Delta t [\beta h_y^{n-1/2}(i+1, j) + h_z^{n-1/2}(i+1/2, j+1/2) \\
&- h_z^{n-1/2}(i+1/2, j-1/2)] / \Delta y \\
d_y^n(i, j+1/2) &= d_y^{n-1}(i, j+1/2) + \\
\Delta t [-\beta h_x^{n-1/2}(i, j+1/2) - h_z^{n-1/2}(i+1/2, j+1/2) \\
&- h_z^{n-1/2}(i-1/2, j+1/2)] / \Delta x \\
d_z^n(i, j) &= d_z^{n-1}(i, j) + \\
[(h_y^{n-1/2}(i+1/2, j) - h_y^{n-1/2}(i-1/2, j)) / \Delta x \\
&- (h_x^{n-1/2}(i, j+1/2) - h_x^{n-1/2}(i, j-1/2)) / \Delta y] \\
\ldots \ldots \ldots \quad (11)
\end{aligned}$$

where the stability factor $s = c\Delta t / \Delta l$, and c is the velocity of light. In these expressions, b and h are normalized such that the characteristic impedance of space is unity. The condition for stability of (10) in free space is^[2]

$$s \leq \sqrt{3} \quad (12)$$

So far, a space-time mesh has been introduced and the Maxwell's equations have been replaced by a finite-difference equations. In the formulation of eigenvalue problems here, only "hard boundaries" (usually represented by perfectly conducting walls) occur. At these boundaries, the tangential electric and the normal magnetic field components are maintained at zero. At the interface of dielectric with air the tensor permittivity $\bar{\epsilon}$ is described by

$$\bar{\epsilon} = \epsilon_0 \begin{bmatrix} (\epsilon_{xx}+1)/2 & \epsilon_{xy}/2 & \epsilon_{xz}/2 \\ \epsilon_{yx}/2 & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx}/2 & \epsilon_{zy}/2 & (\epsilon_{zz}+1)/2 \end{bmatrix} \quad (13)$$

To solve the system of equations (11)

in this mesh, initial values must be assigned first. For rectangular-type structure discussed in this paper, the impulse function is an appropriate choice for eigenvalues of the dominant mode. As n increases, the discrete time functions of e and h fields evolve towards the steady state which is the characteristics of the desired mode in the geometry. The final steady-state field distribution may be calculated by taking the time average of the time domain solution at each mesh point. Thus the steady-state solution is given by

$$F(i_o, j_o) = \sum_n F^n(i_o, j_o) / N \quad (14)$$

In eigenvalue problems, the steady-state solution is a time-harmonic function, from which the eigenvalues can be extracted by discrete Fourier transform,

$$F(f) = \sum_n F^n(i_o, j_o) \exp(-j2\pi snf) \quad (15)$$

III. NUMERICAL SIMULATION RESULTS

In order to simulate the evolution of transient signals propagating in microstrips, two kinds of temporal waveforms, DC square and Gaussian pulses, are examined. Two kinds of widely used substrate materials (GaAs and Sapphire) for microstrip lines are considered in the numerical simulation. For simplicity, the computation is restricted in the case where the principal optical axis of substrate is only oriented in x - y plane as shown in Fig.1, and characterized by the orientation angle θ . In this case the tensor permittivity of the substrate is given by

$$\bar{\epsilon} = \epsilon_0 \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \\ \epsilon_{yx} & \epsilon_{yy} & \\ & & \epsilon_{zz} \end{bmatrix} \quad (16)$$

where ϵ_x , ϵ_y and ϵ_z are the diagonal elements of tensor permittivity when the optical axis coincides with x -axis, i.e., $\theta=0$.

In order to verify the algorithm to compute the eigenvalue $\beta(\omega)$ (or ϵ_{eff}) with the modified 2D FD-TD method, the characteristics of a microstrip with anisotropic substrate treated previously in [3] for $\theta=0$ are

calculated. The obtained results agree well with [3].

Fig.3 shows the dispersed output of DG Gaussian pulse of width $T=50\text{ps}$ for a 1.27cm section of microstrip with a sapphire substrate ($\epsilon_x=11.6$, $\epsilon_y=\epsilon_z=9.4$) for three different orientation angles, while Fig.4 shows those for a square pulse of width $T=1.25\text{ns}$. For comparison, the thickness of substrate and the width of metal strip are chosen to be the same as in [2]. Evidently, the distortion is quite pronounced when the orientation angle is changed. The results are valuable for the design of practical devices, particularly, in mm-wave frequency range.

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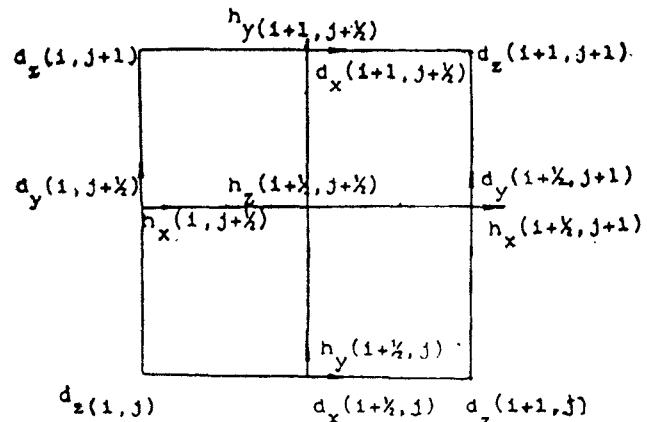


Fig.2 Positions of field components in a difference unit

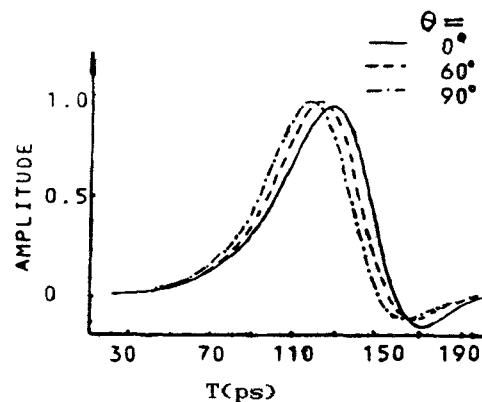


Fig.3 Distortions of Gaussian pulse in microstrip ($L=1.27\text{cm}$, $T=50\text{ps}$)

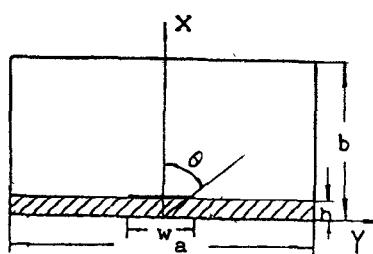


Fig.1 The cross-section of shielded microstrip

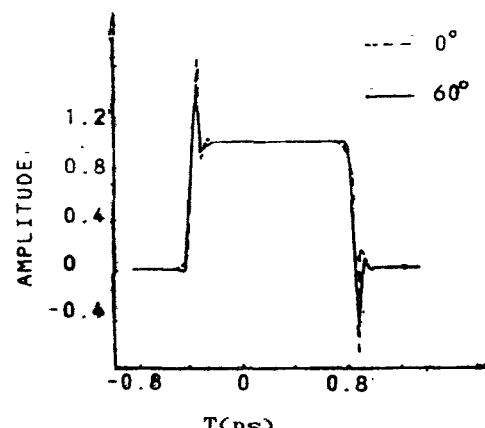


Fig.4 Distortions of square pulse in microstrip ($L=2.54\text{cm}$, $T=1.25\text{ns}$)